

# Distributional Semantics

Magnus Sahlgren

Pavia, 12 September 2012

# Recap

Words-by-regions matrices and LSA

Words-by-words-matrices and HAL

Dependency-based models

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# This lecture

Word space models in the lab

vs.

Word space models in the Real World  
*(dumbing down and scaling up)*

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Corpora (e.g. TASA, ANC, BNC, WaC...)

- Small- to medium-sized
- Static
- Editorial (for the most part)

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*Koptjevskaja-Tamm & Sahlgren, 2012*

	<i>General (BNC)</i>	<i>News (Reuters)</i>	<i>Blogg (Spinn3r)</i>
<b>hot</b>	boiling	warm	castoff
	distilled	inclement	bomsight
	brackish	wintry	warm
	drinking	changeable	scald
	cold	mild	bottled
<b>cold</b>	hot	inclement	cream
	franco-prussian	mild	cube
	boer	warm	rink
	iran-iraq	wintry	floe
	napoleonic	changeable	skating

# Computational Semantics in the Real World

## Data

- Big Data
- Dynamic (streaming) data
- Non-editorial (i.e. noisy)



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## Example 1

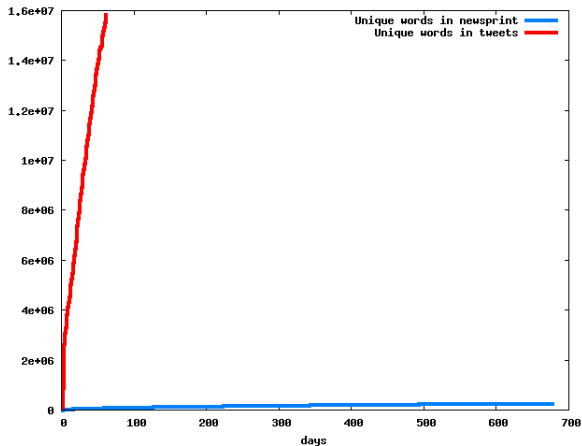
recommend	
recomend	0.972
reccomend	0.968
recommmend	0.941
looove	0.870
loooove	0.863
lurve	0.850
love	0.846
loooooove	0.836

## Example 2

good		bad	
great	0.91	weird	0.86
prefect	0.83	sucky	0.86
perfect	0.83	scary	0.86
pristine	0.81	cool	0.85
stable	0.80	nasty	0.84
grat	0.80	dumb	0.84
fantastic	0.80	sad	0.84
flawless	0.79	lame	0.84
mint	0.79	creepy	0.84
immaculate	0.79	stupid	0.84

# Computational Semantics in the Real World

## New words in New Text



# Computational Semantics in the Real World

Processing cost is critical

Processing dependencies are a liability

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Problem: the huge co-occurrence matrix

Solution: dimension reduction

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## Dimension Reduction

Matrix factorization: Principal Component Analysis (PCA), Singular Value Decomposition (SVD), Independent Component Analysis (ICA), Non-negative Matrix Factorization (NMF), etc.

Random projection:  $A'_{m,k} = A_{m,n}R_{n,k}$   
where  $k \ll n$

Simple statistics (e.g. column variance)

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Another approach: Random Indexing



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# Random Indexing

Designed to be on-line, scalable and efficient

Based on Pentti Kanerva's work on sparse distributed memory

Can be used with documents, words, tuples (and anything else) as contexts

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## Random Indexing

Random Indexing uses several *non-unique* dimensions per context

$$\begin{array}{c} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_m \end{array} \begin{bmatrix} r_1 & r_2 & r_3 & r_4 & r_5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$c_1 = [r_1, r_3]$$

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The risk of randomly selecting the exact same dimensions for two different contexts is negligible (*we'll come back to why*)

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The distributed random representation for a context is called a random **index vector**:

$$a_{ij} = \begin{cases} +1 & \text{with probability } \frac{\epsilon/2}{k} \\ 0 & \text{with probability } \frac{k-\epsilon}{k} \\ -1 & \text{with probability } \frac{\epsilon/2}{k} \end{cases}$$

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- High-dimensional
- Sparse (a small number of active dimensions)
- Ternary (active dimensions have either  $+1$  or  $-1$ )
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Context vectors are accumulated *incrementally*

- Each word has a context vector (initially empty)
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Every time a word occurs, add the region's index vector to the word's context vector

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Word  $w_1$  occurs in document  $d_1$  with index vector:

$[+1, 0, -1, \dots, 0]$

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In a standard co-occurrence matrix, the contexts are *orthogonal* to each other

The distributed random index vectors are *nearly* orthogonal to each other

Selecting dimensions at random in a high-dimensional space will *approximate* orthogonality

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## Lab:

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- Syntagmatic relations by inverse permutations

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A context vector is a sum of (random) index vectors

Word order does not matter when summing vectors:

“the baby **fed** the bear”

$$c(\text{fed}) = r(\text{the}) + r(\text{baby}) + r(\text{the}) + r(\text{bear})$$

“the bear **ate** the baby”

$$c(\text{ate}) = r(\text{the}) + r(\text{bear}) + r(\text{the}) + r(\text{baby})$$

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## Encoding word order

One solution: **circular convolution** (HRR, BEAGLE)

- Context vectors are accumulated from  $n$ -grams (up to 7)
- The  $n$ -grams are bound by circular convolution ( $*$ ) and an auxiliary random vector ( $\Phi$ )

$$c(\text{fed}) = (r(\text{baby}) * \Phi) + (r(\text{the}) * r(\text{baby}) * \Phi) + \dots$$

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Another solution: **permutation** of vector coordinates

- Permutation operation ( $\Pi$ ): rotation of vector coordinates.
- **Direction vectors**: rotate the random index vectors to the left ( $\Pi^{-n}$ ) or to the right ( $\Pi^n$ ) *one* step if a word occurs to the left or right of the focus word (cf. HAL)

$$c(\text{fed}) = \Pi^{-1}r(\text{the}) + \Pi^{-1}r(\text{baby}) + \Pi r(\text{the}) + \Pi r(\text{bear})$$

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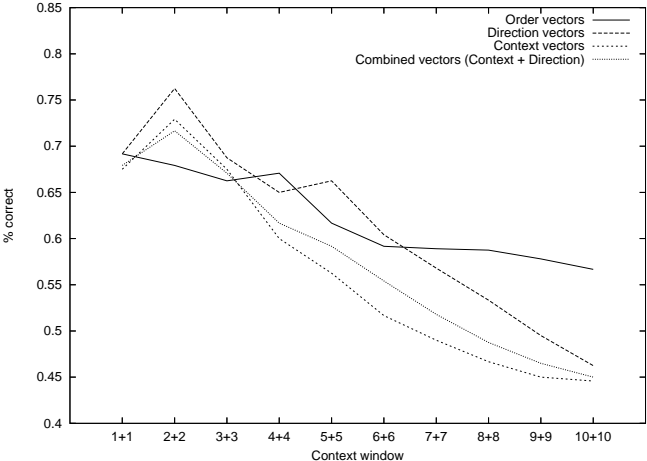
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# Context window size



## Paired-associative memory

Given a trace vector  $t = (x_1 \odot y_1) + (x_2 \odot y_2) + (x_3 \odot y_3) + \dots$

and a probe vector  $x_j$ ,

find the associate  $y_j$  from a set of possible random vectors.

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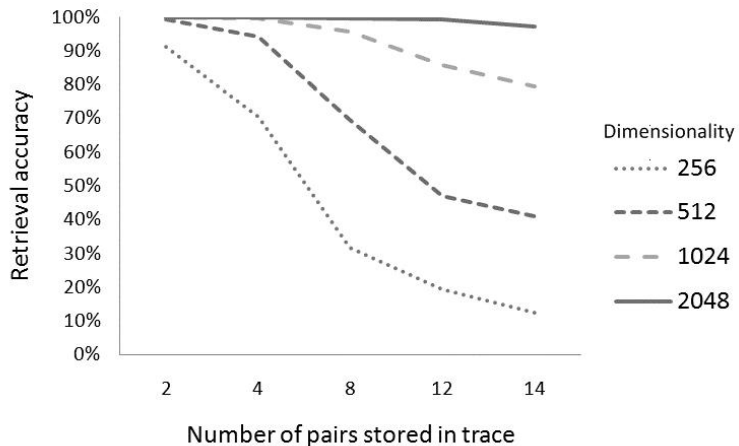
## Paired-associative memory

Given a trace vector  $t = (x_1 \odot y_1) + (x_2 \odot y_2) + (x_3 \odot y_3) + \dots$

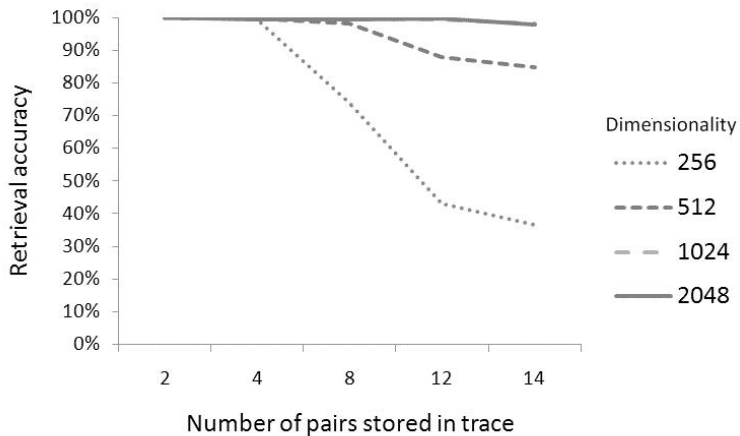
and a probe vector  $x_i$ ,

find the associate  $y_i$  from a set of possible random vectors.

## Circular convolution



## Random permutations



## Results

TOEFL synonym scores, averaged over 3 runs using different initializations of the random vectors.

BEAGLE score: 57.81%

Random permutations:  $\approx$  80%

(Roget's thesaurus: 78.75%, WordNet < 25%  
*Jarmasz & Szpakowicz, 2003*)



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## Order and directional neighbors

Extract frequent left and right neighbors by using the inverse permutation.

Whenever “baby fed” occurs,  $\Pi^{-1} r(\text{baby})$  is added to  $c(\text{fed})$ .

To retrieve “baby” from  $c(\text{fed})$  we will compare  $\Pi c(\text{fed})$  to all *random index vectors*.

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## Examples

### KING

Word before	Word after
luther .24	queen .43
martin .22	england .25
become .17	midas .16
french .14	france .15
dr .13	jr .14

## Examples

### PRESIDENT

Word before		Word after	
vice	.69	roosevelt	.22
become	.23	johnson	.20
elect	.20	nixon	.18
goodway	.09	kennedy	.15
former	.09	lincoln	.15



# Random Permutations

## Lab:

- Build a Random Permutations model

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# Random Indexing

## Summary

- Implicit (built-in) dimension reduction
- Any kind of context
- Permutations to handle structure
- Parallelizable
- Generalizable to tensors

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# Lab

Use GSDM and S-space to:

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- Experiment with Random Permutations

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